Learning Compositional Koopman Operators for Model-Based Control

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Introduction

- Finding an embedding space for a linear approximation of a nonlinear dynamical system enables efficient system identification and control synthesis.
- Previous methods fail to generalize in scenarios with a variable number of objects.

We proposed **Compositional Koopman operators** that
- Use GraphNet to produce object-centric embeddings,
- Use a block-wise linear transition matrix to regularize the shared structure across objects.

Motivating example

Consider a system with N balls connected by linear spring. $\boldsymbol{x}_i \triangleq [x_i, y_i, \dot{x}_i, \dot{y}_i]^T$

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{x}}_1 \\ \dot{\boldsymbol{x}}_2 \\ \vdots \\ \dot{\boldsymbol{x}}_N \end{bmatrix} = \begin{bmatrix} A & B & \cdots & B \\ B & A & \cdots & B \\ \vdots & \vdots & \ddots & \vdots \\ B & B & \cdots & A \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_N \end{bmatrix}$$

Three observations
(1) System state is composed of the state of each object.
(2) Transition matrix has a block-wise substructure.
(3) Same physical interactions share the same block.

Koopman operator theory

- For a nonlinear dynamical system $\boldsymbol{x}^{t+1} = F(\boldsymbol{x}^t)$,
- Identify the nonlinear-to-linear transformations $g : \mathcal{X} \rightarrow \mathbb{R}$
- The Koopman operator $\mathcal{K}$, is a linear transformation on the embedding space:

$$\mathcal{K}g \triangleq g \circ F$$

$$\mathcal{K}g(\boldsymbol{x}^t) = g(F(\boldsymbol{x}^t)) = g(\boldsymbol{x}^{t+1})$$

Compositional Koopman operators

Transition in Koopman space (+ control)

$$g(\boldsymbol{x}^{t+1}) = \boldsymbol{K}g(\boldsymbol{x}^t) + \boldsymbol{L}\boldsymbol{u}^t$$

(1) The Koopman embedding of the system is composed of the Koopman embedding of every objects.

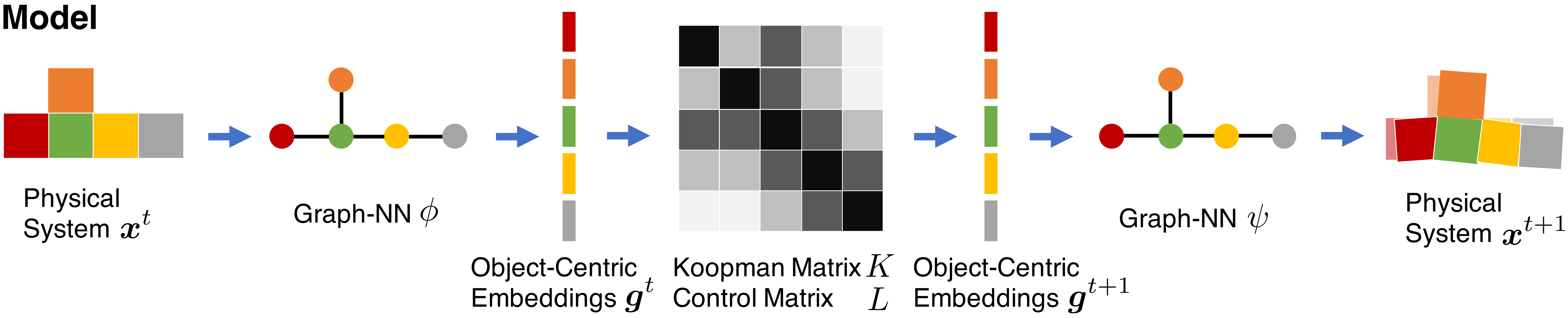
$$\boldsymbol{g}^t = [\boldsymbol{g}_1^{t\,\top}, \cdots, \boldsymbol{g}_N^{t\,\top}]^\top \in \mathbb{R}^{Nm}$$

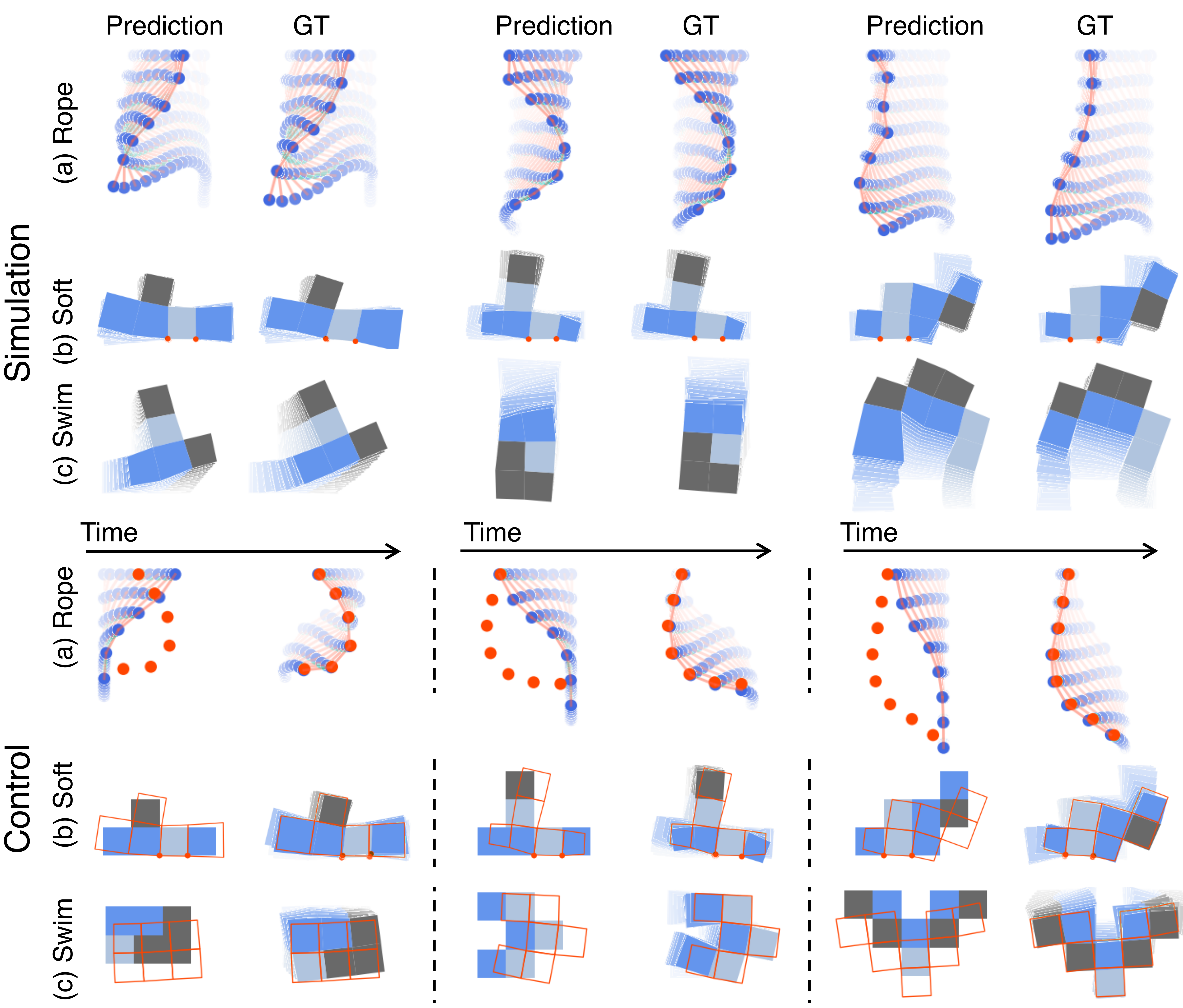
(2) The Koopman matrix has a block-wise structure.

$$\begin{bmatrix} \boldsymbol{g}_1^{t+1} \\ \vdots \\ \boldsymbol{g}_N^{t+1} \end{bmatrix} = \begin{bmatrix} K_{11} & \cdots & K_{1N} \\ \vdots & \ddots & \vdots \\ K_{N1} & \cdots & K_{NN} \end{bmatrix} \begin{bmatrix} \boldsymbol{g}_1^{t} \\ \vdots \\ \boldsymbol{g}_N^{t} \end{bmatrix} + \begin{bmatrix} L_{11} & \cdots & L_{1N} \\ \vdots & \ddots & \vdots \\ L_{N1} & \cdots & L_{NN} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_1^{t} \\ \vdots \\ \boldsymbol{u}_N^{t} \end{bmatrix}$$

(3) The same physical interactions share the same sub-block.

Model





Loss Function

- Auto-encoding loss
- Forward prediction loss
- Metric loss

$$\mathcal{L} = \mathcal{L}_{\text{ae}} + \mathcal{L}_{\text{pred}} + \mathcal{L}_{\text{metric}}$$

System Identification

- Least-square Regression

$$\min_{K,L} \| K\boldsymbol{g}^{1:T-1} + L\tilde{\boldsymbol{u}} - \boldsymbol{g}^{2:T} \|_2$$

Control Synthesis

- Quadratic Programming

Website

http://koopman.csail.mit.edu/

References

[1] Lusch et al. Deep learning for universal linear embeddings of nonlinear dynamics. **Nature communications 2018**
[2] Yunzhu Li et al. Propagation Networks for Model-Based Control Under Partial Observation. **ICRA 2019**
[3] Peter W. Battaglia et al. Interaction Networks for Learning about Objects, Relations and Physics. **NeurIPS 2016**